AAL, Third homework set

You have to submit 3 of the following 4 exercises.

1. Let G be a finite group, $N \triangleleft G$ and p a prime. Suppose that M is a normal p-complement in G. Show that $N \cap M$ is a normal p-complement in N and $N \cdot M/N$ is a normal p-complement in G/N.

2. Let G be a finite group. The probability that two elements of G commute equals to $\frac{c}{|G|}$, where c is the number of conjugacy classes.

3. Let A be an abelian subgroup of G (such that $|G : A|$ is finite). Let $g \in G$ and $b \in N_G(A)$. Show that $\tau_{G \to A}(g)$ commutes with b, where $\tau_{G \to A}$ denotes the transfer homomorphism from G to A.

4. Let V be an infinite dimensional vector space over $\mathbb{F}_p$, which is the finite field of $p > 2$ elements. Let $<,>$ denote a skew-symmetric bilinear function from V^2 to $\mathbb{F}_p$ (i.e $< u, v >= - < v, u >$.). Assume further that if v is a vector in V such that $< v, u >=0$ for every $u \in V$, then $v = 0$. Let G be $\{(v, a) \mid v \in V \;\; a \in \mathbb{F}_p\}$. We define a binary operation on G:

$$(u, a)(v, b) := (u + v, a + b + < u, v >)$$

   (a) Determine $Z(G)$.

   (b) Determine G'.

   (The group G is called the infinite extraspecial p-group.)