

AAL, Second homework set

1. Let H and K be subgroups of the group G . Prove that $|HK| = \frac{|H||K|}{|H \cap K|}$.
2. Prove that a group of order 616 cannot be simple.
3. Let G be a group. The subgroup $D = \{(g, g) \mid g \in G\}$ of $G \times G$ is called the *diagonal subgroup* of G .
 - Prove that D is a normal subgroup in $G \times G$ if and only if G is abelian
 - A subgroup M of a group H is called a *maximal subgroup* if $M \subsetneq H$ and there is no subgroup K satisfying $M \subsetneq K \subsetneq H$. Prove that D is a maximal subgroup of $G \times G$ if and only if G is a simple group, i.e. it has no proper normal subgroup.