AAL, Ex. 3.

1. Let H be a finite index subgroup of G. Prove that H contains a finite index normal subgroup of G. More precisely, if $|G : H| = n$, then there exists $N \leq H$ such that $|G : N| \leq n!$ and $N \trianglelefteq G$.

2. Prove that every left-coset is a right coset (of not necessarily the same subgroup) as well.

3. Prove that the inverse of a left coset is a right coset of the same subgroup. The inverse of a subset A of G is $\{g^{-1} \mid g \in A\}$.

4. Consider the action of the dihedral group D_n on the vertices. Determine all values of n for which the action is primitive.

5. Let G be a primitive permutation group and let $N \neq \{1\}$ be a normal subgroup of G. Prove that N is transitive.

6. Let $A \leqslant B \leqslant G$ (such that $|G : A|$ is finite) and assume that B is abelian. Prove that (for all $g \in G$) $\tau_{G \to A}(g) = \tau_{B \to A}(\tau_{G \to B}(g))$.