AAL, Ex. 2.

1. Let G be a k-transitive permutation group acting on a finite set X of size n. Then $n(n-1)\dots(n-k+1) \mid |G|$.

2. Let $G \leq S_{20}$ having an orbits of length 11 and 19. Does it follow that G has an element fixing exactly 11 points.

3. Let G be a group of order 312. Prove that G is not simple.

4. Let G be a group of order $4k+2$, $k \geq 1$. Prove that G is not simple.

5. - Prove that $|GL(n,p)| = (p^n-1)(p^n-p)\cdots\dots\cdot(p^n-p^{n-1})$.
   - Prove that the set U of upper triangular matrices with 1's on the diagonal forms a Sylow p-subgroup of $GL(n,p)$.

6. (Burnside's lemma)
   A bracelet is made by sliding coloured beads to a string and tying its ends. How many different bracelets can you make with 6 red and 6 blue beads? Note that you can put the bracelet into your wrist in two possible ways and you can rotate it.